1. a) Is it possible that the sum of all the positive divisors of two different natural numbers are equal?
b) Is it possible that the product of all the positive divisors of two different natural numbers are equal?

2. How many ways can you fill a table of size $n \times n$ with integers such that each cell contains the total number of even numbers in its row and column other than itself?
Two tables are different if they differ in at least one cell.

3. At least how many non-zero real numbers do we have to select such that every one of them can be written as a sum of 2019 other selected numbers and
a) the selected numbers are not necessarily different?
b) the selected numbers are pairwise different?

4. Suppose that you are given the foot of the altitude from vertex A of a scalene triangle ABC, the midpoint of the arc with endpoints B and C, not containing A of the circumscribed circle of ABC, and also a third point P. Construct the triangle from these three points if P is the
a) orthocenter
b) centroid
c) incenter
of the triangle.

5. Let p be prime and $k > 1$ be a divisor of $p - 1$. Show that if a polynomial of degree k with integer coefficients attains every possible value modulo p (that is, $0, 1, \ldots, p - 1$) at integer inputs then its leading coefficient must be divisible by p.
Note: the leading coefficient of a polynomial of degree d is the coefficient of the x^d term.

Please write all the solutions on separate pages. Make sure to write the name of your team and the category on every paper.
Each problem is worth 12 points. The duration of the contest is 180 minutes. Good luck!

the organizers of the XIII.Dürer Competition