E
category

XVI. DÜRER COMPETITION

First round:
18th November 2022.

MATHEMATICS
PROBLEMS

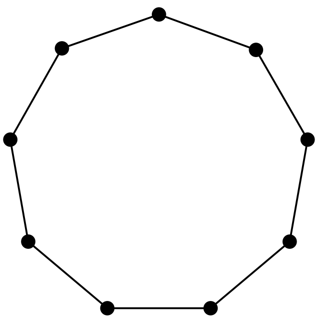
---

1. A group of students play the following game: they are counting one by one from 00 to 99 taking turns, but instead of every number they only say one of its digits. (The numbers in order are 00, 01, 02, ..., meaning that one-digit numbers are regarded as two-digit numbers with a first digit 0.) One way of starting the counting could be for example 0, 1, 2, 0, 4, 0, 6, 7, 8, 9, 1, 1, 2, 1, 1, 5, 6, 1, 8, 1, 0, 2 etc. When they reach 99, the counting restarts from 00. At some point Csongor enters the room and after listening to the counting for a while, he discovers that he is able to tell what number the counting is at. How many digits has Csongor heard at least?

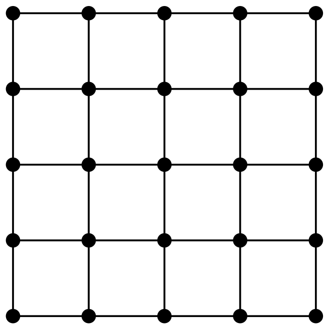
2. In a Greek village of 100 inhabitants in the beginning there lived 12 Olympians and 88 humans, they were the first generation. The Olympians are 100% gods while humans are 0% gods. In each generation they formed 50 couples and each couple had 2 children, who form the next generation (also consisting of 100 members). From the second generation onwards, every person's percentage of godness is the average of the percentages of his/her parents' percentages. (For example the children of 25% and 12.5% gods are 18.75% gods.)
a) Which is the earliest generation in which it is possible that there are equally many 100% gods as 0% gods?
b) At most how many members of the fifth generation are at least 25% gods?

3. Pitagoras drew some points in the plane and connected some of these with segments. Now Tortillagoras wants to write a positive integer next to every point, such that one number divides another number if and only if these numbers are written next to points that Pitagoras has connected. Can Tortillagoras do this for the following drawings?

a)



b)



In part b), vertices in the same row or column but not adjacent are not connected.

4. Let k be a circle with diameter AB and centre O. Let C be an arbitrary point on the circle different from A and B. Let D be the point for which O, B, D and C (in this order) are the four vertices of a parallelogram. Let E be the intersection of the line BD and the circle k, and let F be the orthocenter of the triangle OAC. Prove that the points O, D, E, C, F lie on a circle.

5. Let $n \geq 3$ be an integer. Timi thought of n different real numbers and then wrote down the numbers which she could produce as the product of two different numbers she had in mind. At most how many different positive prime numbers did she write down (depending on n)?

Please write all the solutions on separate sheets. Make sure to write the name of your team and the category on every paper.
Each problem is worth 12 points. The duration of the contest is 180 minutes. Good luck!

the organizers of the XVI. Dürer Competition