$\mathbf{E^+}$ category

XVI. DÜRER COMPETITION

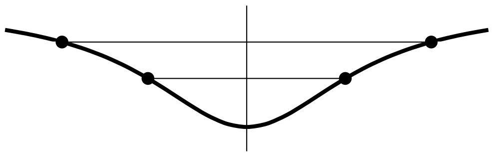
Final round
10-12 February, 2023

MATHEMATICS
RELAY ROUND
PROBLEMS

E$^+$-1. Nüx has three moira daughters, whose ages are three distinct prime numbers, and the sum of their squares is also a prime number. What is the age of the youngest moira? (3 points)

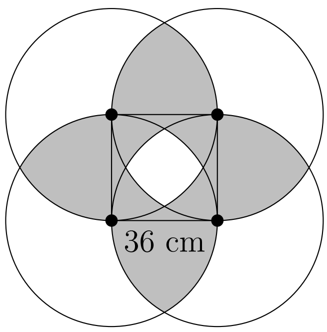
E$^+$-2. The area of a rectangle is 64 cm^2, and the radius of its circumscribed circle is 7 cm. What is the perimeter of the rectangle in centimetres? (3 points)

E$^+$-3. Hapi, the god of the annual flooding of the Nile is preparing for this year's flooding. The shape of the channel of the Nile can be described by the function $y = \frac{-1000}{x^2+100}$ where the x and y coordinates are in metres. The depth of the river is 5 metres now. Hapi plans to increase the water level by 3 metres. How many metres wide will the river be after the flooding? *The depth of the river is always measured at its deepest point.* (3 points)



E$^+$-4. In Eldorado a year has 20 months, and each month has 20 days. One day Brigi asked Adél who lives in Eldorado what day her birthday is. Adél answered that she is only going to tell her the product of the month and the day in her birthday. (For example, if she was born on the 19th day of the 4th month, she would say $4 \cdot 19 = 76$.) From this, Brigi was able to tell Adél's birthday. Based on this information, how many days of the year can be Adél's birthday? (3 points)

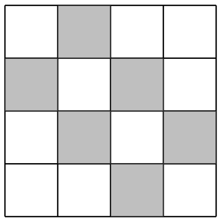
E$^+$-5. King Minos divided his rectangular island of Crete between his 3 sons as follows: he built a wall along one diagonal of the island and gave one half of the island to his eldest son. Then, in the remaining triangular area, from the right-angled vertex he built a wall perpendicular to the other wall. Of the two areas thus obtained, the larger was given to the middle son and the smaller to the youngest. Each of the three sons had the largest possible square palace built on his own land. How many times is the area of the eldest son's palace larger than the area of the youngest son's palace if the side lengths of the island are 30 m and 210 m? (4 points)



E$^+$-6. Archimedes drew a square with side length 36 cm into the sand and he also drew a circle of radius 36 cm around each vertex of the square. If the total area of the grey parts is $n \cdot \pi$ cm^2, what is the value of n?
Do not disturb my circles! (4 points)

E$^+$-7. One day Mnemosyne decided to colour all natural numbers in increasing order. She coloured 0, 1 and 2 in brown, and her favourite number, 3, in gold. From then on, for any number whose sum of digits (in the decimal system) was a golden number less than the number itself, she coloured it gold, but coloured the rest of the numbers brown. How many four-digit numbers were coloured gold by Mnemosyne?
The set of natural numbers includes 0. (4 points)

E$^+$-8. Zoli wants to fill the given 4×4 table with the digits 1, 2, 3 and 4, such that in every row and column, and also in the diagonal going from the top left cell to the bottom right, each digit appears exactly once. What is the highest possible value of the sum of the digits in the six grey cells? (4 points)



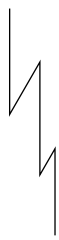
							1
			0			1	1
	1						
1				0			1
				0	0		
						0	
			0				
				1		0	

E$^+$-9. The *binary sudoku* is a puzzle in which a table should be filled with digits 0 and 1 such that in each row and column, the number of 0s is equal to the number of 1s. Furthermore, there cannot exist three adjacent cells in a row or in a column such that they have the same digit written in them. Solving the given binary sudoku, what is the sum of the numbers in the two diagonals? (5 points)

E$^+$-10. Marvin really likes pancakes, so he asked his grandma to make pancakes for him. Every time Grandma sends pancakes, she sends a package of 32. When Marvin is in the mood for pancakes, he eats half of the pancakes he has. Marvin ate 157 pancakes for lunch today. At least how many times has Grandma sent pancakes to Marvin so far? *Marvin does not necessarily eat an integer number of pancakes at once, and he is in the mood for pancakes at most once a day.* (5 points)

E$^+$-11. A country has 2023 cities and there are flights between these cities. Each flight connects two cities in both directions. We know that you can get from any city to any other using these flights, and from each city there are flights to at most 4 other cities. What is the maximum possible number of cities in the country from which there is a flight to only one city? (5 points)

E$^+$-12. Zeus's lightning is made of a copper rod of length 60 by bending it 4 times in alternating directions so that the angle between two adjacent parts is always 60°. What is the minimum value of the square of the distance between the two endpoints of the lightning?
All five segments of the lightning lie in the same plane.
(5 points)



E$^+$-13. Csongi bought a 12-sided convex polygon-shaped pizza. The pizza has no interior point with three or more distinct diagonals passing through it. Áron wants to cut the pizza along 3 diagonals so that exactly 6 pieces of pizza are created. In how many ways can he do this? *Two ways of slicing are different if one of them has a cut line that the other does not have.* (6 points)

E$^+$-14. For the Dürer final results announcement, four loudspeakers are used to provide sound in the hall. However, there are only two sockets in the wall from which the power comes. To solve the problem, Ádám got two extension cords and two power strips. One plug can be plugged into an extension cord, and two plugs can be plugged into a power strip. Gábor, in his haste before the announcement of the results, quickly plugs the 8 plugs into the 8 holes. Every possible way of plugging has the same probability, and it is also possible for Gábor to plug something into itself. What is the probability that all 4 speakers will have sound at the results announcement? *For the solution, give the sum of the numerator and the denominator in the simplified form of the probability. A speaker sounds when it is plugged directly or indirectly into the wall.* (6 points)

E$^+$-15. What is the biggest positive integer which divides $p^4 - q^4$ for all primes p and q greater than 10? (6 points)

E$^+$-16. What is the remainder of $2025\hat{}(2024\hat{}(2022\hat{}(2021\hat{}(2020\hat{}\ldots\hat{}(2\hat{}1)\ldots)))))$ when it is divided by 2023?

Here ^ is the exponential symbol, for example $2\hat{}(3\hat{}2) = 2\hat{}9 = 512$. *The power tower contains the integers from* 2025 *to* 1 *exactly once, except that the number* 2023 *is missing.* (6 points)