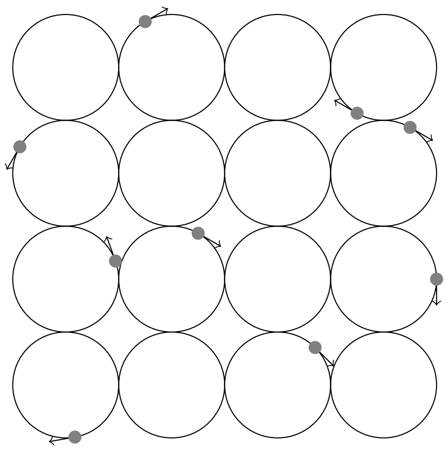
E$^+$-1. A number is called a *duck number* if by reading it backwards, we obtain a greater number than the original one. How many 14-digit duck numbers can be written using only the digits 1 and 2?
For example, 37145 *is a duck number because* $54173 > 37145$.

(3 points)

E$^+$-2. A swordfish has a special relationship with the number 8, so its favourite numbers are exactly those three-digit numbers for which the number itself, the sum of its digits and the product of its digits are all divisible by 8, but none the number's digits are divisible by 8. What is the sum of the swordfish's favourite numbers?

(3 points)

E$^+$-3. On the circles of the diagram there are 9 ants walking around, all of them with the same constant speed and without turning around or stopping. Whenever an ant reaches a point of tangency of two circles, it can choose which of the two circles it continues on, but without turning around. An ant cannot turn around even when it switches to another circle. Each time two ants meet, one of them climbs over the other and if they have not met before, one of them shouts once. What is the maximum number of times that the ants shouted if their initial positions and directions are as shown on the diagram?
We know that it is not possible that two ants meet while travelling in the same direction.



(3 points)

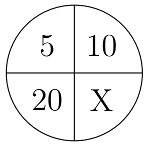
E$^+$-4. Let N be the smallest positive integer such that the half of N is a perfect square, the third of N is a perfect cube and the fifth of N is a perfect fifth power. How many positive divisors does N have?

(3 points)

E$^+$-5. Dorka wrote some different positive integers on a piece of paper with the following property: for any integer $2 \leq K \leq 100$ that Lili thinks of, she can find some distinct numbers (at least one) among the ones written by Dorka whose product is exactly K. At least how many numbers did Dorka write on the paper?

(4 points)

E$^+$-6. Captain Morgan threw 9 darts aiming at the board composed of four sectors, depicted on the diagram. If a dart hit the board, then the Captain received points equal to the number in the sector that the dart hit. If the dart did not hit the board, the Captain received no points for that dart. How many distinct positive integers could be in the place of the X, if the Captain scored a total of 175 points?



Captain Morgan didn't hit the border of any sector. (4 points)

E$^+$-7. At an individual maths competition there were 4 students participating from each of the two classes 12.a and 12.b. We know that the students got 8 different total scores, meaning that their overall positions were also different. Simply by looking at the 8 positions, their teacher was able to conclude that the sum of scores of the students from class 12.b is greater than the sum from 12.a. What is the number of the possible orders of the eight students?

For any two students, the one having more points will have a higher position. If two students have the same score then their position is the same. The teacher does not even know the maximum possible score for the competition. (4 points)

E$^+$-8. We glued together a $3 \times 5 \times 5$ cuboid from small cubes of the same size, where the middle cube is red and the rest of the cubes are yellow, each with a different shade.

Anita removed some cubes (at least one) in such a way that the remaining object was a cuboid, with no visible red on its surface. How many different ways could the resulting cuboid have looked like?

Anita could have also removed cubes from the bottom of the cuboid, and all the faces of the remaining cuboid were visible. (4 points)

E$^+$-9. Gabi asked Beni when his birthday was. Beni mysteriously replied that he is only telling the value of m^d, where m and d respectively represent the month and day of his date of birth. From this, Gabi has not yet been able to determine when Beni's birthday was. Based on this, how many possible days of the year are there on which Beni could have been born?

(5 points)

E$^+$-10. The circle k with centre A has a radius of 14 units, and B is a point on k. The circle ℓ is tangent to segment AB in its midpoint and is also tangent to k. Let C be the point for which the incircle of ABC is ℓ. How many units is the perimeter of triangle ABC? (5 points)

E$^+$-11. Fill in the grid with the digits $1, 2, 3, 4, 5, 6$ such that every row and column contains each digit exactly once. Furthermore, the numbers beside the grid indicate the highest absolute difference between any two consecutive digits in the row/column. **The answer is the four digit number obtained by reading the digits in the grey colored squares from top to bottom.** (5 points)

	3		3	1		
5						
2						1
5						
3					2	

E$^+$-12. The altitudes of an acute triangle are 585, 600 and 936 units long. How many units is the perimeter of the triangle?

(5 points)

E$^+$-13. Six villages, Arc, Bock, Chap, Deck, Et and Fuse are situated along a road in this order. There is a bus service between any pair of consecutive villages. For the five services, buses leave every 5, 7, 9, 11 and 12 minutes, but we don't know which pair of consecutive villages corresponds to which frequency (between any two given villages, the frequency is constant). Today, Alex, Aron and Ben arrived at different times to the bus station in Arc: Alex arrived at 12:00, Aron at 12:01 and Ben at 13:00. They all aimed to get to Fuse; Alex arrived at the bus station in Fuse at 14:20, but Aron arrived only at 15:00. How many minutes elapsed between the arrivals of Aron and Ben in Fuse?

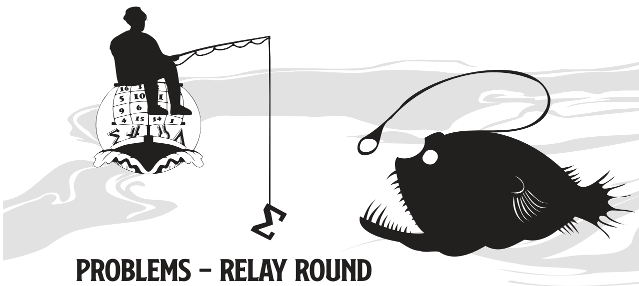
The buses come on time, they leave and arrive in a whole minute, buses between the same pair of villages always take the same time, transfer doesn't take any time, and they all took the earliest possible bus at each station.

(6 points)

E$^+$-14. What is the value of $\sum\limits_{k=1}^{17} \frac{1}{k(k+1)(k+2)(k+3)}$?

As an answer, give the numerator of the simplest form of the fraction.

(6 points)



E$^+$-15. Benjamin thought of a real number x and told Timi the value of $\lfloor x^3 \rfloor$. We know from Timi that this is a positive integer not larger than 100 and that by knowing the value of $\lfloor x^3 \rfloor$, the value of $\lfloor x^2 \rfloor$ cannot be determined uniquely. Let A denote the number of possible values that Timi could have heard. After this Benjamin thought of a real number y and told Timi the value of $\lfloor y^4 \rfloor$. Timi also says that this second value is a positive integer not larger than 100 and that by knowing the value of $\lfloor y^4 \rfloor$, the value of $\lfloor y^3 \rfloor$ cannot be determined uniquely. Let B denote the number of possible values that Timi could have heard as the second value. What is the value of $A + B$?
The notation $\lfloor r \rfloor$ denotes the integer part of the real number r, which is the largest integer not greater than r.

(6 points)

E$^+$-16. A positive integer n is called *infernal* if $\dfrac{1! \cdot 2! \cdot \ldots \cdot 100!}{n!}$ is an integer and also a square of an integer. What is the sum of the infernal numbers?

(6 points)