



1. In a competition, there are a total of 20 participants across several different categories, and each participant competes in exactly one category. The numbers 2, 3, ..., 21 are assigned as bib numbers before the competition begins, with each participant receiving exactly one number. The organizer wants to distribute the bib numbers in such a way that no matter how the results turn out within each category, for any two categories, there will always be one person from each podium whose bib numbers are not relatively prime.

a) Prove that the organizer can distribute the bib numbers in this way if there are 5 categories, each with 4 participants.

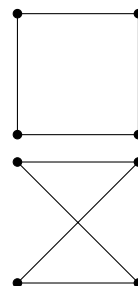
b) Can the organizer distribute the bib numbers in this way if there are 4 categories, each with 5 participants?

c) Does such a distribution of bib numbers exist if there are two categories, each with 10 participants?

On each podium, there are always three participants from the same category. The organizer knows which participant belongs to which category. Two different integers are relatively prime if their greatest common divisor is 1.

For example, in part b), the following distribution would not work: if the first category has bib numbers 4, 11, 14, 20, 21, and the second category has 3, 9, 10, 12, 15. In this case, it is not true that, regardless of the results, there will be one person from each podium whose bib numbers are not relatively prime. For instance, if the podium in the first category consists of participants with bib numbers 4, 11, and 14, and in the second category 3, 9, and 15, then any choice of one number from the first podium and one from the second will always be relatively prime.

2. In an elven city, a sociologist wants to map the residents' acquaintance network. For privacy reasons, the network does not include names. It is known that for any three residents A , B , and C , the following holds: if A and B do not know each other, and B and C do not know each other, then A and C do not know each other either. Furthermore, if two residents know each other, then they have the same number of acquaintances. For which numbers of residents is the number of possible networks odd?



Acquaintance is mutual. Two networks are considered identical if, based solely on the networks, it is possible that exactly the same pairs of residents know each other. For example, the two networks shown in the figure are considered identical.

3. By a *partition* of a regular n -gon we mean drawing $n - 3$ diagonals that do not intersect each other. A partition is called *tricky* if among the resulting $n - 2$ triangles, exactly two are isosceles. For which integers $n \geq 4$ does a regular n -gon admit a tricky partition?

4. Let ABC be a triangle and let D and E be two points on line BC such that the order of the four points is D, B, C, E . Let X denote the intersection point of the circumcircles of triangles ABC and ADE which is different from A . The line parallel to AD through B intersects line AC in point F , and the line parallel to AE through C intersects line AB in point G . Similarly the line parallel to XD through B intersects line XC in point H , and the line parallel to XE through C intersects line XB in point I . Show that points F, G, H, I lie on a line.

5. The role-playing game D.Ü.R.E.R. is played by n players sitting around a circular table. The role of the storyteller is determined as follows. Initially, the oldest player holds a fair die. In one step, the player who currently has the die rolls it, and if the result is d , the die is passed d times one seat to the left. This step is repeated k times. The player who holds the die at the end becomes the storyteller. Determine all values of n for which there exists a k such that the above method of selection is fair.

A selection method is fair if every player has the same probability of becoming the storyteller.

Please write all the solutions on separate pages. Make sure to write the name of your team and the category on every paper.

Each problem is worth 12 points. The duration of the contest is 180 minutes. Good luck!

the organizers of the XIX. Dürer Competition