E+
category

First round (2025. November 14.) **Problems**

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1. Tyrion and Littlefinger play a game that begins with 2026 empty boxes numbered from 0 to 2025. In each round, the next player in turn chooses a box with index i that does not yet contain a number, and then the other player chooses an integer a_i and places it in the box. The players alternate choosing boxes, with Tyrion going first. The game ends when every box contains a number. If $x = 5202$ is a solution for the equation $a_{2025}x^{2025} + a_{2024}x^{2024} + \ldots + a_1x + a_0 = 0$, then Tyrion wins, otherwise Littlefinger wins. Who has a winning strategy?
Throughout the game, both players know the contents of all boxes.

2. Let ABC be a triangle and let D and E be two points on line BC such that the order of the four points is D, B, C, E. Let X denote the intersection point of the circumcircles of triangles ABC and ADE which is different from A. The line parallel to AD through B intersects line AC in point F, and the line parallel to AE through C intersects line AB in point G. Similarly the line parallel to XD through B intersects line XC in point H, and the line parallel to XE through C intersects line XB in point I. Show that points F, G, H, I lie on a line.

3. A goblin has eaten all positive integers n for which n divides the sum of squares of every positive integer less than n that is also coprime to n. Prove that there exist 2025 consecutive positive integers, all of which have been eaten by the goblin!

4. Let p be a fixed prime number. We call a nonempty finite subset H of the plane *p-fantastic* if it is possible to write positive integers on the points of H such that all of the following three conditions hold:

- Not all points of H lie on a single line.
- There exists a point in H on which a number not divisible by p is written.
- If a line contains at least two points of H, then the sum of the numbers written on the points on the line is divisible by p.

Determine the minimum size of a p-fantastic set of points as a function of p.
Exactly one number is written on each of the points of H, and there are no numbers on other points.

5. Is there a bounded infinite sequence of real numbers $a_1, a_2, \ldots$ for which $|a_i - a_j| > \frac{1}{|i-j|}$ holds for all $i \neq j$?
An infinite sequence of real numbers is called bounded if there exists a real number M for which $|a_i| \leq M$ holds for all i.

Please write all the solutions on separate pages. Make sure to write the name of your team and the category on every paper.
Each problem is worth 12 points. The duration of the contest is 180 minutes. Good luck!

the organizers of the XIX. Dürer Competition