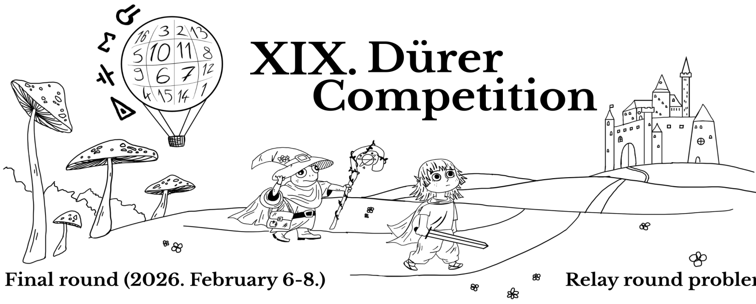


XIX. Dürer Competition

E+ category

Final round (2026. February 6-8.) — Relay round problems

E$^+$-1. How many two-digit positive integers are there which are equal to the sum of their digits summed with the product of their digits? (3 points)

E$^+$-2. Ricky the baby rook lives on a chess board which is infinite in all directions. Ricky walks in his sleep every night, during which he walks to a neighbouring square, but he does not remember which one. Each noon the queen of the chess board tells him how many times he has visited so far each of the four squares neighbouring the square he is currently in. She always tells these numbers in a random order, therefore Ricky can never know which of the numbers is referring to which square. Ricky is happy on days when he can be sure that he is on a square that he has been on before. Which is the first day that Ricky can be happy on?

The first time Ricky sleepwalks is on the night between the first and second day. During the day he does not change place and he always remembers what he has been told on the previous days.

(3 points)

E$^+$-3. If H is a subset of the integer numbers of size 10, then for all of its non-empty subsets we take the sum of its elements. Let $P(H)$ denote the product of these 1023 numbers. What is the sum of all the prime numbers which divide $P(H)$ for all possible sets H?

All elements of a set are different. (3 points)

E$^+$-4. On the International Mathematical Olympiad there are 3 problems on both days. The problems are across 4 topics: algebra, geometry, number theory and combinatorics. The organisers initially select 3 easy, 3 medium and 2 hard problems from each topic. Then from these the jury forms the final list of problems in a way such that the following conditions hold:

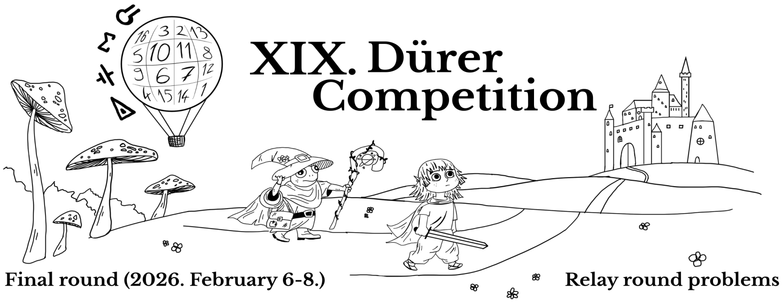
- Problems 1 and 4 are easy, 2 and 5 are medium, 3 and 6 are hard.
- All four topics should be covered in problems 1, 2, 4 and 5.
- No topics should appear more than once on a given day.

How many different ordered problem lists can the jury create if they decided that the first problem will be an algebra?

Problems 1, 2, 3 are on the first day and problems 4, 5, 6 are on the second day. (3 points)

E$^+$-5. Given are a circle k with centre O and radius 1 and a line e tangent to O. Let f be a line which is perpendicular to e and intersects k in two different points. Let the intersection points of f with e and k respectively be E, A and B, lying on f in this order. Let the intersection point of lines e and BO be I. Let line g be the tangent through I to k, which is different to e, and let its point of tangency be J. Let us assume that g is perpendicular to e. Furthermore let the point of intersection of lines BI and AJ be H and let the length of the segment AH be x. What is the value of $(x^2-4)^2$?

(4 points)



E+
category

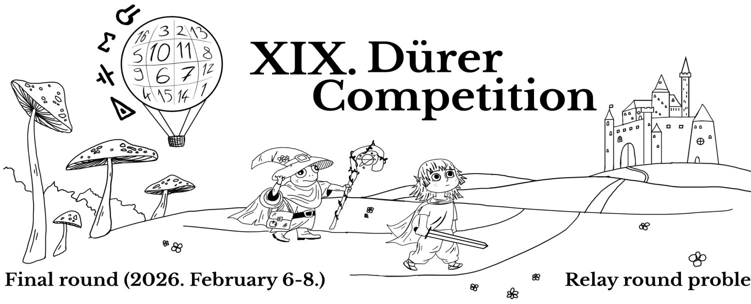
E$^+$-6. Moriarty professor throws with a regular dice infinitely many times. What is the probability that he throws a 6 before throwing odd numbers three times? **Answer with the sum of the numerator and the denominator in the simplified form of the fraction.**
The three odd numbers do not need to be thrown consecutively. (4 points)

E$^+$-7. A wizard has one solution with concentration 1 in a blue bottle, and another solution with concentration 0 in a red bottle. With the help of two bottles of different colours, the wizard can magically create a new solution in an empty red or blue bottle, which has concentration the average of the concentrations of the used solutions. At most how much can be the sum of the concentrations in the blue bottles of the wizard after 15 magical creations? **Answer with the sum of the numerator and the denominator in the simplified form of the fraction.**
The solutions cannot be poured from one bottle to another. When the wizard makes a new solution, the colours and contents of the used bottles don't change. (4 points)

E$^+$-8. We have drawn the lines of all sides and diagonals of a regular nonagon (9-gon). How many equilateral triangles are formed by these lines? (4 points)

E$^+$-9. A group of 12 thieves is holding a meeting at a round table. Every thief either engages in a mutual conversation with one of his neighbours, or just stares ahead without talking with anyone. In how many different configurations can the conversations take place?
Two configurations are considered different if there are two people who are talking with each other in one configuration but not in the other. (5 points)

E$^+$-10. While Juliet was secretly reading a book, Romeo became quite curious about the numbers of the two pages her book is currently opened at. He asked her a few questions, from which he got to know a few things: among the two page numbers, there was at least one palindrome, at least one perfect power, at least one number with every digit being even, at least one number containing the digit 5 at least once, and at least one number greater than or equal to 10. From all of this information, Romeo could uniquely determine the two page numbers. What is the maximum possible value for the page number of the book?
The pages of Juliet's book are numbered in increasing order from 1 to n, and Romeo knows the value of n. However, Romeo does not know whether the even-numbered pages are on the left or on the right. A number is called palindrome if you get the exact same number when you write its digits in the reverse order. A positive integer is called a perfect power if it can be written in the form a^b, where a and b are positive integers, and $b \geq 2$. (5 points)



XIX. Dürer Competition

E+ category

Final round (2026. February 6-8.) Relay round problems

E⁺-11. Let $t(n)$ denote the sum of the digits of the number n. What is the number of digits in the smallest positive integer n such that $t(2n) = \frac{2}{9}t(n)$? (5 points)

E⁺-12. Let $T_k(q(x))$ denote the polynomial which is obtained from the polynomial $q(x)$ by only taking its terms of degree at most k and deleting the rest of its terms. Let $p(x)$ be a polynomial of degree 2026 with integer coefficients, such that $T_i(p(x))$ divides $p(x)$ for every integer $0 \leq i \leq 2026$. What is the maximum possible number of different coefficients that $p(x)$ can have?
For example, $T_3\left(x^4 + 0x^3 + 2x^2 + 0x - \frac{1}{2}\right) = 0x^3 + 2x^2 + 0x - \frac{1}{2}$. We say that the polynomial $T_i(p(x))$ divides the polynomial $p(x)$ if there exists a polynomial $t(x)$ with integer coefficients such that $T_i(p(x)) \cdot t(x) = p(x)$. (5 points)

E⁺-13. Benedek wrote two rational numbers on the board. Their product is an integer, and when writing both numbers in their simplified mixed fraction form, the six integers appearing in these two forms are $1, 2, 3, 4, 5, 6$ in some order. What is the number of possible pairs of rational numbers that Benedek could have written on the board? (The order of the two rational numbers does not matter.)
The rational number $\frac{p}{q}$ has simplified mixed fraction form $x + \frac{y}{z}$, where x is the integer part of the number, and $\frac{y}{z}$ is the simplified form of the fractional part of the number. (6 points)

E⁺-14. Fill the squares of the 6×6 grid with the digits $1, 2, 3, 4, 5, 6$ such that in every row and every column, each digit appears exactly once. The numbers outside the table denote the largest possible sum of two adjacent numbers in that row or column. What is the product of the four numbers in the corners of the 6×6 grid? (6 points)

	9	11	8	9		10
10		6			5	1
7						
		3			1	6

E⁺-15. We would like to enter the numbers $1, 2, \ldots, 9$ into the nine squares below, in some order. The inequalities must be correct. In each square, we must write exactly one number, and each number must be used exactly once. In how many ways can we do this?

$$\square < \square > \square < \square > \square < \square > \square < \square > \square$$

(6 points)

E⁺-16. What is the number of distinct trapezoids that have four integer-length sides and a total perimeter of 85 units?
Two trapezoids are considered distinct if they are not congruent. Trapezoid is a quadrilateral that has at least one pair of parallel sides. (6 points)